Daniel Müllner Spring 2012

Applied Algebraic Topology, Homework 3

Due: Tuesday, May 22 in class

1. Let $S \subseteq \mathbb{R}^d$ be a finite set of points. Recall that Čech(r) and Alpha(r) are the Čech and alpha complexes for radius $r \geq 0$, and let Del(S) denote the Delaunay complex associated with S. Give a proof or a counterexample to the following two inclusions:

i) $\mathrm{Alpha}(r) \subseteq \check{\mathrm{C}}\mathrm{ech}(r) \cap \mathrm{Del}(S)$

ii) $\mathrm{Alpha}(r) \supseteq \check{\mathrm{C}}\mathrm{ech}(r) \cap \mathrm{Del}(S)$

(6 points)

2. Let $L \subset \mathbb{R}^n$ be a finite subset. Prove that $\mathrm{Del}(L) \subseteq W(\mathbb{R}^n, L, 0)$. Give an example where equality does not hold.

(4 points)

3. i) Let $0 \to V_1 \to \ldots \to V_n \to 0$ be a finite exact sequence of finite-dimensional vector spaces. Prove that $\sum_i (-1)^i \dim V_i = 0$.

ii) Let K be a simplicial complex and U, V subcomplexes that cover K. Suppose that all homology groups of U, V and K (for some coefficient field k) are finite-dimensional, and only finitely many are nonzero. Prove that $\chi(K) = \chi(U) + \chi(V) - \chi(U \cap V)$.

(10 points)